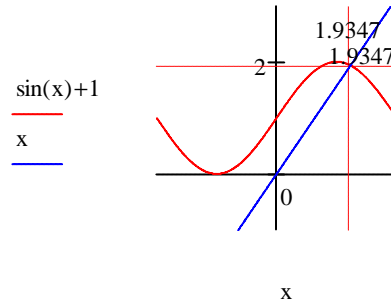
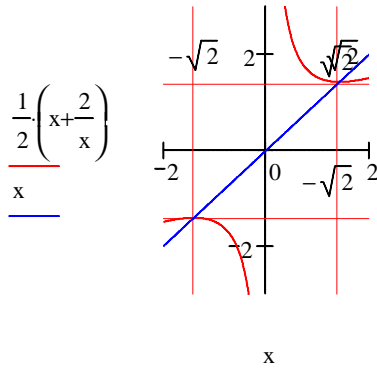


实验20

迭代法求方程的根

Using iteration method solve equation $x=f(x)$



Solve equation $x = \sin(x) + 1$

Compute square root of $X=2$

$$X := 2 \quad y_0 := 1.5 \quad j := 0..4$$

$$y_{j+1} := \frac{1}{2} \left(y_j + \frac{X}{y_j} \right)$$

$$y = \begin{pmatrix} 1.5 \\ 1.41666666666667 \\ 1.41421568627451 \\ 1.41421356237469 \\ 1.41421356237309 \\ 1.41421356237309 \end{pmatrix}$$

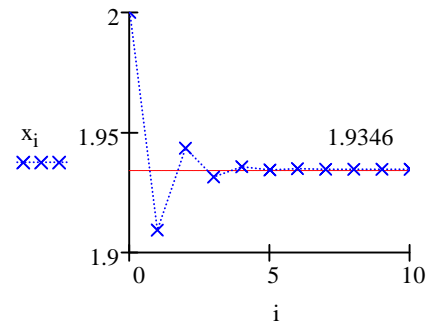
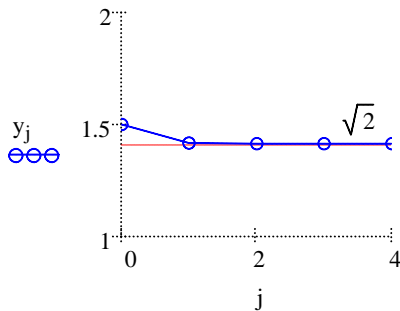
$$\sqrt{2} = 1.4142135623731$$

$$\sqrt{2} = 1.414213562373095048801688724209698078570$$

$$x_0 := 2 \quad i := 0..10$$

$$x_{i+1} := \sin(x_i) + 1$$

	0
0	2
1	1.90929742682568
2	1.94325347008953
3	1.9314359910797
4	1.93567129497719
5	1.93416838388212
6	1.93470361617592
7	1.93451324569203
8	1.93458098701091
9	1.93455688586229
10	1.93456546111101
11	1.93456241007837



Convergence is very fast!

```

iteration(f,a,n) :=
  for i ∈ 0..n-1
  |
  |   xi ← a
  |   yi ← f(xi)
  |   a ← yi
  |
  | augment(x,y)

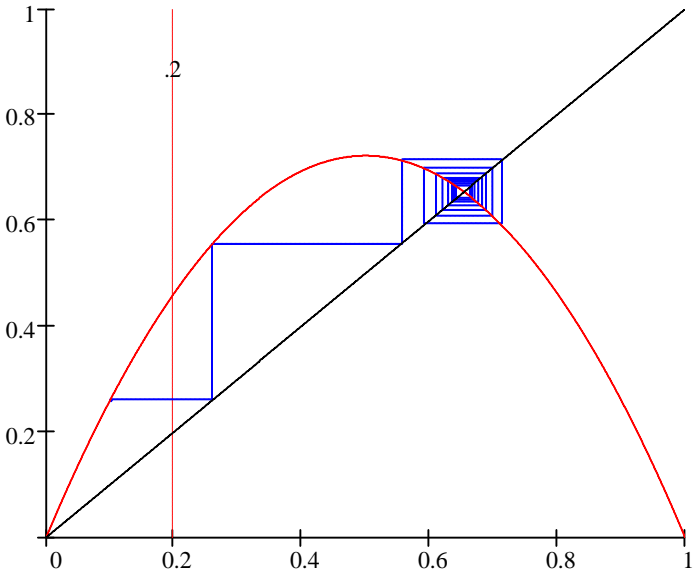
```

$$f(x) := 2.89 \cdot x \cdot (1 - x)$$

	0	1
0	0.5	0.723
1	0.723	0.579
2	0.579	0.704
3	0.704	0.602
4	0.602	0.692
5	0.692	0.615
6	0.615	0.684
7	0.684	0.625
8	0.625	0.678

$$\text{iteration}(f,0.5,25) =$$

$$Y := \text{iteration}(f,.1,18)$$



$$y := 0.8$$

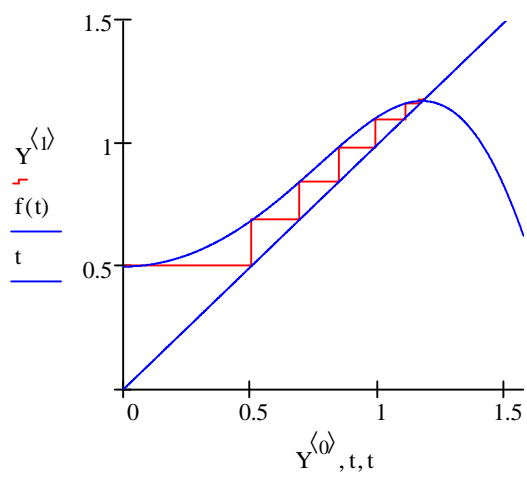
$$\text{root}(f(y) - y, y) = 0.654$$

$$f(x) := .5 \cos(x) + \sin(x^2)$$

	0	1
0	0	0.5
1	0.5	0.686
2	0.686	0.84
3	0.84	0.983
4	0.983	1.1
5	1.1	1.162
6	1.162	1.175
7	1.175	1.175
8	1.175	1.175

$$\text{iteration}(f,0,10) =$$

$$Y := \text{iteration}(f,0,20)$$



$x := 1$

$\text{root}(f(x) - x, x) = 1.1745$